**Traffic Flows**

* **Problem Statement**

A traffic congestion will bring various negative influences to a county. In Ghana, traffic congestion had caused a delay in its economic development. This is due to the lost productive time caused by the long traffic jams. Besides that, the high congestion level jeopardized the environment as the poisonous gasses were emitted into the air. According to Addison (2016), some countries in Ghana were even suffering traffic congestion for three to eight hours daily.

Thus, partial differential Equations (PDEs) are used based on the Lighthill and Whitham (1955) and Richard (1956) (LWR) model to manage the vehicle traffic flow between the traffic lights in the cities of Ghana (from Kwame Nkrumah Circle to Adabraka) effectively. Furthermore, the method of characteristics is also used to solve the systems of PDEs.

* **Methodology**

In the thesis, the traffic flow is discussed by using a macroscopic approach which is based on the LWR model. It shows the fundamental relationship among the three traffic flow parameters (density, flow, and speed) as below:

(1)

where *q* is flow (*veh/s*), *v* is speed (*m/s*), and *k* is density (*veh/m*).

According to Greenshields (1935), the relationship between *v* and *k* can be written by

(2)

where *vmax* is some maximal velocity as a property of the road, and *kmax* is the maximal bumper to bumper density.

When multiplying both sides of equation (2) with *k*,

(3)

Based on the LWR model, the traffic flow can be written in a one-dimensional continuity equation

(4)

where *k(x, t)* is the density (in number of vehicles per unit length of a road at time *t*), and *v* is the traffic stream velocity.

After doing partial equation, the LWR model can be expressed as

(5)

* **Results and Discussion**

The data was collected and analyzed. The equation (3) is then used to show the result of regression for the flow versus density:

(6)

**Non-linear homogeneous PDE**

From equation (6), , the maximum point can be found.

By substituting into ,

while the velocity at the maximum flow is

From the LWR model (equation (5)),

(7)

Since the solution is *k = k(x, t)*, taken the derivative with respect to *t*,

(8)

Compare equation (7) with equation (8),

So, on a certain curve *x(t)* the solution *k(x, t)* of equation (6) is a constant.

**Non-linear inhomogeneous PDE**

Let ,

When integrated,

When ,

Hence,

When , then

where *k* is density, *β* is a constant and *t* is time. This represents the particular solution of the general solution.

**Method of Characteristic**

Apply the method of characteristics to

along the initial condition *k(x, 0) = 93.87x* at *t = 0*.

Thus, making *x* the subject in

as the characteristic which starts at *x = x0* when *t = 0*.

Substituting *k(x0, 0) = 93.87 x0* into the above equation

The solution of non-linear homogeneous is obtained by making *x0* the subject of the characteristic equation. Thus

Since *k(x, t) = F(x0)*, the solution of the non-linear homogeneous PDE is given as

where *k(x, t)* is the density (in number of vehicles per unit length of a road at time *t*), *x* is the position and *t* is time. This represents the complementary function of the general solution.

* **Conclusion**

The maximum flow on the road network was *0.511 veh/s*.

The maximum density was *53.505 veh/m*.

The maximum density was *0.0096 m/s*.

To prevent traffic congestion, traffic flow should be designed to move at a density corresponding to maximum traffic flow.

**References**

Addison, E. A. (2016). *Modelling vehicle traffic flow with partial differential equation.* (Thesis, Presbyterian University College). Retrieved from <https://www.researchgate.net/publication/316088790_MODELLING_VEHICLE_TRAFFIC_FLOW_WITH_PARTIAL_DIFFERENTIAL_EQUATIONS>

China Engineering Consultants, Inc. (2016). *Research of macroscopic traffic flow models* (Report No. 05922). Retrieved from <http://www.ceci.org.tw/file.ashx?id=8b04aebf-c160-477c-b6e3-63fa9a3ba096>